CBCS SCHEME

USN | | | | | | | | | | 17MAT11

First Semester B.E. Degree Examination, July/August 2021
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions.

1 a. Find the n^{th} derivative of $\cos x \cos 3x \cos 5x$. **(06 Marks)**
b. If $\tan y = x$, then prove that
$(1 + x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$ **(07 Marks)**
c. Derive the angle between radius vector and the tangent. **(07 Marks)**

2 a. If $y = a\cos(\log x) + b\sin(\log x)$ then show that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2 + 1)y_n = 0$. **(06 Marks)**
b. Find the pedal equation of the polar curve $\frac{2a}{r} = (1+\cos\theta)$. **(07 Marks)**
c. Find the radius of curvature for the folium of De-Cartes $x^3 + y^3 = 3axy$ at $(3a/2, 3a/2)$. **(07 Marks)**

3 a. Expand $e^{\sin x}$ using Maclaurin's theorem upto the term containing x^4. **(06 Marks)**
b. If $U = \log\sqrt{x^2 + y^2 + z^2}$ show that $(x^2 + y^2 + z^2)\,[U_{xx} + U_{yy} + U_{zz}] = 1$. **(07 Marks)**
c. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$ show that $J\left(\frac{x,y,z}{r,\theta,\phi}\right) = r^2\sin\theta$. **(07 Marks)**

4 a. Expand $\log(1 + \cos x)$ by Maclaurin's series upto the term containing x^4. **(06 Marks)**
b. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ **(07 Marks)**
c. If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$ **(07 Marks)**

5 a. A particle moves along the curve $x = t^3 - 4t$, $y = t^2 + 4t$, $z = 8t^2 - 3t^3$ where 't' denotes time. Find the component of its acceleration at $t = 2$ along the tangent. **(06 Marks)**
b. Find $\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. **(07 Marks)**
c. Prove that $\operatorname{div}(\operatorname{curl}\vec{A}) = 0$. **(07 Marks)**

6 a. A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$. Find the velocity and acceleration at any time 't' and also their magnitudes at $t = 0$. **(06 Marks)**
b. If $\vec{A} = xz^3 i - 2x^2yz j + 2yz^4 k$ find $\nabla.\vec{A}$ and $\nabla.(\nabla\times\vec{A})$ at the point $(1, -1, 1)$. **(07 Marks)**
c. Find the directional derivatives of $\phi = \frac{xz}{x^2 + y^2}$ at $(1, -1, 1)$ along $i - 2j + k$. **(07 Marks)**

7 a. Obtain the reduction formula for

$\int \cos^n x$ and hence evaluate $\int_0^{\pi/2} \cos^n x dx$ **(06 Marks)**

b. Solve $(x^2 + y^2 + x)\,dx + xy\,dy = 0$ **(07 Marks)**

c. Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$ where λ is a constant. **(07 Marks)**

8 a. Evaluate $\int_0^{\pi/6} \cos^4 3x \sin^2 6x$ using reduction formula. **(06 Marks)**

b. Solve $\frac{dy}{dx} + \frac{x + 3y - 4}{3x + 9y - 2} = 0$ **(07 Marks)**

c. Find the orthogonal trajectories of the family of curves $\left(r + \frac{k^2}{r}\right)\cos\theta = a$, 'a' being parameter. **(07 Marks)**

9 a. Solve the following system of equations by Gauss-Seidel method to obtain the final solution correct to three decimal places.
$x + y + 54z = 110$, $27x + 6y - z = 85$, $6x + 15y + 2z = 72$. **(06 Marks)**

b. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to diagonal form. **(07 Marks)**

c. Find the numerically largest eigen value and the corresponding eigen vector of the following matrix using power method

$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ taking initial approximation as $[1 \quad 0.8 \quad -0.8]^T$ **(07 Marks)**

10 a. Solve by Gauss elimination method $2x + y + 4z = 12$, $4x + 11y - z = 33$, $8x - 3y + 2z = 20$. **(06 Marks)**

b. Diagonalize the matrix $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. **(07 Marks)**

c. Using Rayleigh's power method find the numerically largest eigen value and the corresponding eigen vector of the matrix

$A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ by taking $[1 \quad 0 \quad 0]^T$ as the initial vector. **(07 Marks)**

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